Exam Advanced Logic

April 11th, 2012

Instructions:

- Put you name and student number on the first page.
- Put your name on subsequent pages as well.
- Do not use pencil (or an erasable pen) or a red pen to make your exam.
- Motivate all your answers.
- If you want to inspect your exam after it has been graded, you can do so by making an appointment with Barteld Kooi.
- Please fill in the anonymous course evaluation.

Good luck!

1. **Induction (10 pt)** Consider the sublanguage $\mathscr{L}_{\to}$ of the language of propositional logic with $\to$ as its only logical operator. (So without $\neg$, $\wedge$, $\vee$ and $\leftrightarrow$).

   (a) Give an inductive definition of $\mathscr{L}_{\to}$.

   (b) Prove by induction that the number of left parentheses "(" is equal to the number of right parentheses ")" in each formula of $\mathscr{L}_{\to}$.

2. **Three-valued logics (10 pt)** Determine whether the following holds in Ł_3 using a truth table.

   $$\neg((p \wedge q) \supset r) \models \neg r \supset (\neg \vee \neg q)$$

   NB: write down the *whole* truth table. Do not forget to draw a conclusion from the truth table.

3. **FDE tableau (10 pt)** By constructing a suitable tableau determine whether the following is valid in **FDE**. If the inference is invalid, provide a counter model.

   $$p \wedge \neg p, \neg(q \vee \neg q) \vdash (\neg p \vee q) \wedge \neg(\neg p \vee q)$$

   NB: Do not forget to draw a conclusion from the tableau.

4. **Fuzzy logic (10 pt)** Determine whether the following holds in $\text{Ł}_{\aleph}$ (where $D = \{1\}$). If so, show that if the premises have value 1, so does the conclusion. If not, provide a counter-model.

   $$p \to (q \vee r), \neg r \models \neg q \to \neg p$$

5. **Basic modal tableau (10 pt)** By constructing a suitable tableau determine whether the following is valid in K. If the inference is invalid, provide a counter model.

   $$\Diamond((p \wedge \Diamond(p \wedge \Diamond q)) \vee (q \wedge \Diamond(q \wedge \Diamond p))) \vdash \Diamond\Diamond\Diamond(p \vee q)$$

   NB: Do not forget to draw a conclusion from the tableau.

6. **Normal modal tableau (10 pt)** By constructing a suitable tableau determine whether the following is valid in $K^t_{\tau\phi\beta}$. If the inference is invalid, provide a counter model.

$$\langle F\rangle p \vdash_{K^t_{\tau\phi\beta}} [F](\langle P\rangle p \vee \langle F\rangle q)$$

NB: Do not forget to draw a conclusion from the tableau.

7. **First-order modal tableau (10 pt)** By constructing a suitable tableau determine whether the following is valid in CK_η. If the inference is invalid, provide a counter model.

$$\forall x \exists y \Box (Px \supset Qy) \vdash_{CK_\eta} \exists x \Box \forall y (Px \supset Qy)$$

NB: Do not forget to draw a conclusion from the tableau.

8. **Soundness and completeness (10pt)** Suppose there is an open complete branch b in a modal tableau for K. Show that b is faithful to the interpretation $\mathcal{I} = \langle W, R, v\rangle$ induced by b. In your proof you may use the Soundness Lemma for K and the Completeness Lemma for K.

9. **Default logic (10 pt)** Consider the folowing set of default rules:

$$D = \left\{ d_1 = \frac{P(x) : R(x)}{S(x) \wedge K(x)}, \quad d_2 = \frac{S(x) : Q(x)}{L(x)}, \quad d_3 = \frac{P(x) : K(x)}{\neg Q(x)} \right\},$$

and initial set of facts:

$$W = \{P(j), \forall x(L(x) \to \neg R(x))\}.$$

You only need to apply the default rules to the relevant constant j. Recall that a formula φ is a *sceptic consequence* if and only if φ is true in every extension of (W, D), while it is a *credulous consequence (goedgelovig gevolg)* if and only if φ is true in some extension of (W, D).

(a) Draw the process tree of this default theory.

(b) Is $\neg R(j)$ a sceptic consequence of this theory?

(c) Is $\neg Q(j)$ a credulous consequence of this theory?